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# 1.  INTRODUCTION

## 1.1  The Problem of Control in Decentralized Organizations

In recent years, much emphasis has been placed on decentralized decision-making in large organizations. This is due to the fact that very sophisticated decision aids, large amounts of decision information, and the need for quick response have made it impossible to refer all critical decisions to the top levels of the hierarchy. This thrust toward decentralized control has led to several new problems, however. One such issue concerns the need for higher hierarchical levels to monitor the decisions made at lower levels, and, if necessary, intervene in the event of unsatisfactory performance. In general, three steps are involved in any approach to this problem: first, the organization and its elements must be mathematically modeled; second, the associated information structure or flow must be determined; and final'., the appropriate monitoring and control strategy must be selected for that organization. Much work has been done in the first two areas, particularly in the modeling of strategic and tactical military structures [1]. But the third area, at least in the military application, still relies heavily on traditional command structures, channels, and leadership theory [2].

Recently, several papers have been published which formalize the notions of incentives in organizations whose participants have different objective functions, mainly due to competition between members ([3] and [4]). An important result of this work is that virtual cooperation between the members can be induced by selecting an appropriate control strategy, while the members continue to maximize their own objectives.

INCENTIVE CONTROL FOR A
TACTICAL AIR CONTROL SYSTEM

BY

SCOTT VAN TONNINGEN

B.S., United States Air Force Academy, 1976

JUL 1 1 1984

**A**

THESIS

Submitted in partial fulfillment of the requirements
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in the Graduate College of the
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Urbana, Illinois

## ACKNOWLEDGEMENTS

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# 1. INTRODUCTION

## 1.1 The Problem of Control in Decentralized Organizations

In recent years, much emphasis has been placed on decentralized decision-making in large organizations. This is due to the fact that very sophisticated decision aids, large amounts of decision information, and the need for quick response have made it impossible to refer all critical decisions to the top levels of the hierarchy. This thrust toward decentralized control has led to several new problems, however. One such issue concerns the need for higher hierarchical levels to monitor the decisions made at lower levels, and, if necessary, intervene in the event of unsatisfactory performance. In general, three steps are involved in any approach to this problem: first, the organization and its elements must be mathematically modeled; second, the associated information structure or flow must be determined; and finally, the appropriate monitoring and control strategy must be selected for that organization. Much work has been done in the first two areas, particularly in the modeling of strategic and tactical military structures [1]. But the third area, at least in the military application, still relies heavily on traditional command structures, channels, and leadership theory [2].

Recently, several papers have been published which formalize the notions of incentives in organizations whose participants have different objective functions, mainly due to competition between members ([3] and [4]). An important result of this work is that virtual cooperation between the members can be induced by selecting an appropriate control strategy, while the members continue to maximize their own objectives.

## 1.2  Outline of a Proposed Solution

It is the purpose of this thesis to demonstrate how incentive theory may be applied to the problem of hierarchical control within the Tactical Air Control System (TACS).  In Chapter 2, we supply the necessary background in incentive theory, as it applies to organizations and their elements whose operations can be modeled by objective functions.  Also in this Chapter, we briefly define the structure of a typical TACS.  Then, in Chapter 3, we provide a very simple model of the basic building block of the TACS, the Modular Control Element (MCE).  Finally, in Chapters 4 and 5, we propose two methods for applying incentive control to the leader-follower relationship between the Tactical Air Control Center (TACC) and the MCE.  The first method, the piece-wise linear approach, provides total insensitivity to variations in an internal parameter of the MCE model, while the second method, the quadratic approach, provides only minimum sensitivity but is more analytically appealing.  Each of these chapters also contains a numerical example to help illustrate the concepts.

## 2.  BACKGROUND

### 2.1  Incentive Theory in Organizations

The problem that is addressed in incentive control is that of ensuring that the members of a particular organization behave in a manner that optimizes the performance of the entire organization [3].  We will assume that the participants in the organization each have an individual objective function which must be optimized, denoted $J_i$.  There is also an overall organizational objective function, $J_o$, which must be optimized by the organizational leader.

Suppose that each organizational element has an objective function of the form

$$J_i = f(\beta_i, r, n),\qquad(1)$$

where r is an independent variable, $\beta_i$ is a characteristic, internal, constant parameter of the element, and n is a parameter which depends upon external data.  Suppose further that the overall organizational objective function is a linear combination of the elemental objective functions:

$$J_o = a_1 J_1 + a_2 J_2 + \ldots + a_N J_N\qquad(2)$$

Finally, we assume that each $J_i$ is a convex function in r, so that each element is responsible for __maximizing__ its individual objective by finding the corresponding $r_i$, denoted $r_i^*$, such that

$$J_{i_{max}} = f(\beta_i, n, r_i^*)$$

$$r_i^* = \{r: \ dJ_i/dr = 0\}\qquad(3)$$

It is apparent from Equation (2) that if each element maximizes its own $J_i$, then $J_o$ will also be maximized.

The problem arises when a participating element, which we shall call follower, has a different perception of its own objective function than does the organizational manager, whom we denote leader. One interpretation of this problem is that the follower has a different perception of its internal parameter, $\beta_i$, although the structure of its function remains unchanged. Suppose the leader assumes that the follower has parameter $\beta'$, while the follower thinks he is characterized by a slightly different constant, $\beta$ (the subscript i is dropped from now on). If the maximization in Equation (3) depends upon $\beta$, then the follower will arrive at a different $r^*$ than the value calculated by the leader, denoted $r^{*\prime}$. Once the leader observes this discrepancy, he must consider two issues:

(1) Is the discrepancy large enough to worry about?

(2) Is the leader's perception of $\beta$ more appropriate?

If the answer to both questions is affirmative, the leader may want to intervene using an incentive strategy [3].

An incentive strategy is a function, announced by the leader to the follower, which induces the follower to maximize the leader's objective function, while still maximizing its own objective. In this case, it would induce the follower to behave as if its internal parameter were $\beta'$, instead of $\beta$, thus maximizing $J_1$ at $r^{*\prime}$. In order to implement any type of incentive control, however, the leader must have at least partial control over one of the objective function parameters, and this parameter must subsequently affect the output. Referring to Equation (1), we assume the leader can partially control the external variable n.

Now the leader constructs n as a function of r, the independent variable. This function, denoted $\hat{n}$, is announced to the follower, who can incorporate $\hat{n}$ into its objective function before maximizing. If $\hat{n}$ is constructed properly, the follower's new objective function will have its maximum at r*', even though the follower continues to use $\beta$, rather than $\beta$'. This effect is illustrated in Figure 1. In Figure 1(a), the control $\hat{n}$ is a constant (incentive not used) so the follower, using $\beta$, maximizes at r*, instead of the desired r*'. In Figure 1(b), $\hat{n}$ is a function of r, so constructed that the maximum now occurs at r*', the leader's maximum point, though MCE still assumes $\beta$.



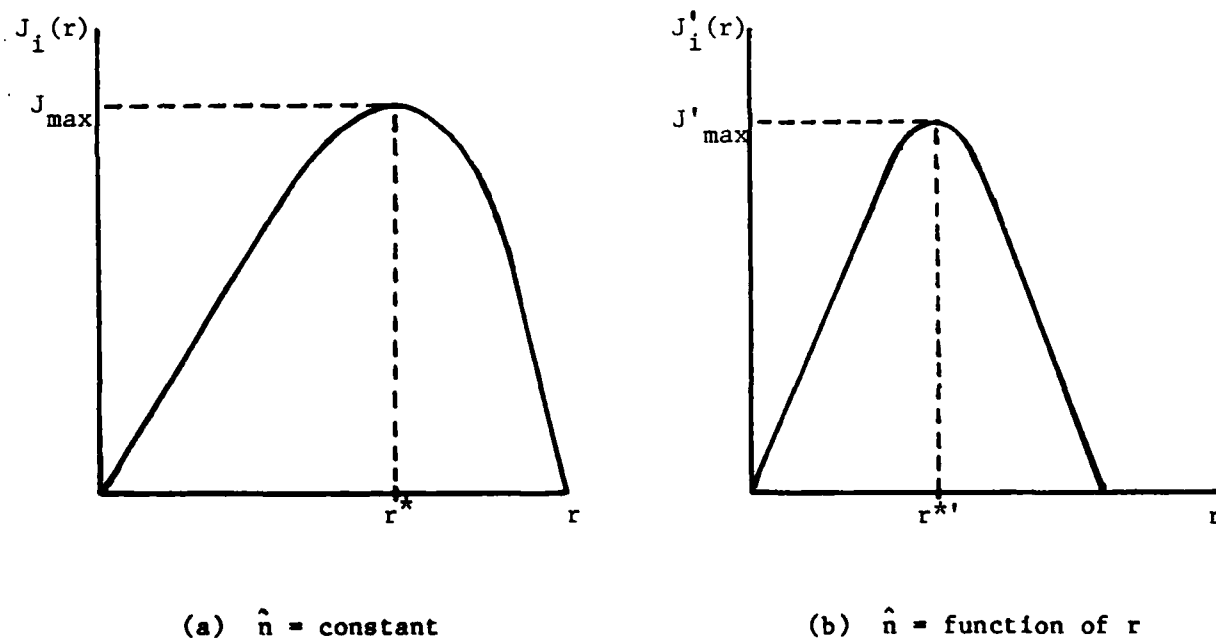(a)  $\hat{n}$ = constant          (b)  $\hat{n}$ = function of r

Figure 1.  Typical follower objective function

Note that $J'_{max}$ may not be the same as $J_{max}$, but this is not important since the leader is assuming that $\beta$, which the follower perceives, is not

accurate anyway. The important fact is that with the control, the follower

maximizes at $r*'$, which corresponds to $\beta'$.

An additional desired quality of this control scheme is robustness.

That is, the incentive strategy should be designed so that the follower

will maximize at $r*'$ for a range of $\beta$, since the leader will not, in

general, know the exact $\beta$ which the follower will assume.

## 2.2. The Tactical Air Control System (TACS)

The TACS is a hierarchical, command, control, and communications ($C^3$)

network, whose purpose is the detection, identification, tracking and

control of aircraft in a defined geographical area. Under the distributed,

decision-making concept, each level of the TACS is tasked with certain func-

tions, such as surveillance, and has the distributed data and hardware

available to make accurate and timely decisions. The general structure of

TACS is very flexible, to allow tailoring to a specific geographical

threat, but for this thesis, can be modeled as in Figure 2:
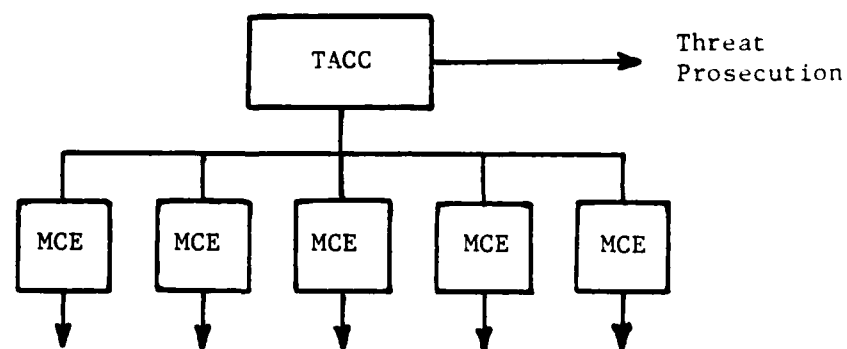


(More MCE units at lower levels as required)

Figure 2. Typical TACS organization

The Tactical Air Control Center (TACC) is assigned overall control of the system and, under present TACS organization, has a lateral information flow relationship with the threat prosecution assets. The Modular Control Elements (MCE) are the distributed decision-makers of the system, and can assume various levels in the TACS hierarchy, as depicted in Figure 2. The focus of this thesis will be on the direct relationship between TACC and one subordinate MCE. The results of this treatment will be equally applicable to all subordinate MCEs on an individual basis.

## 3. THE MODEL

We mentioned in Chapter 1 that extensive work has already been done in the area of modeling the components of $C^3$ organizations and associated information structures. Therefore, no attempt is made in this thesis to provide a realistic, detailed model. Instead, a very simplistic model is developed, so that the concepts of incentive control may be clearly presented and easily understood.

### 3.1 General Structure of the Follower (MCE)

The first task in developing a suitable model for the MCE (and ultimately, its objective function) is to distinguish between two subsystems at work within MCE:

(1) The command subsystem, which is MCE commander or the command structure.

(2) The execution subsystem, which carries out the MCE function, at the direction of the commander.

The command subsystem, for the purposes of this thesis, simply has the job of maximizing the MCE objective function, J, given n and determining a value of $\beta$. Using terminology developed in [5], the parameter n will be the number of "customers" which MCE must process. The output of the MCE commander is r, which we call the "customer processing rate." This dictated rate then becomes the input to the execution subsystem, which is also influenced by n and $\beta$, but simply reacts to the input r. Finally, the output of the execution subsystem is the objective function, J, evaluated at r.

It is for TACC, then, to sample the output, r, of the command subsystem, and observe J, to determine if MCE needs control. TACC, in

general, does not have access to $\beta$, which is MCE commander's perception of

the internal parameter, but must generate an estimate, $\beta'$. Finally, if

TACC does decide to intervene, the resulting n which he computes is

announced to the command subsystem but enforced, if necessary, at the exe-

cution sybsystem. Figure 3 shows a block diagram of the complete system.
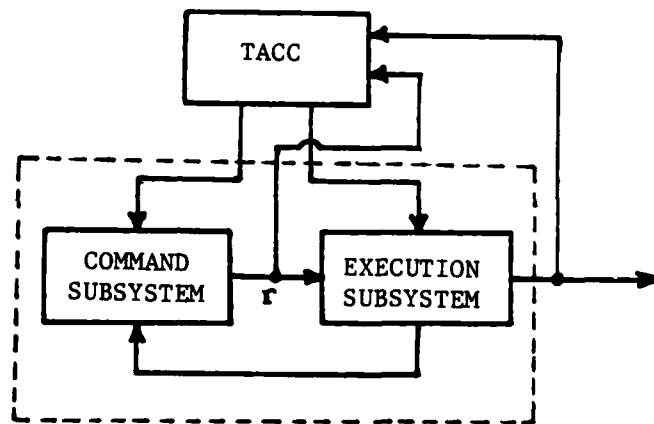


Figure 3. Block diagram of TACC-MCE interaction

## 3.2 Modeling of MCE

The next step in the process, as it applies to TACS, is to model the

MCE execution subsystem. One of the more documented methods of modeling

human system elements is the transfer function approach [6]. It is par-

ticularly suited to our problem since r, the MCE independent rate variable,

is easily viewed as a frequency domain variable (operations/time unit).

The main problem encountered in the transfer function approach to

human modeling is that it is almost universally accepted that human com-

ponents are non-linear, while the transfer function is a linear analysis

tool. The transition from a non-linear model to a linear one is provided

by [7], where linear and non-linear components of the human system, in the frequency domain, are combined as

$$J(r) = H(r) \cdot X(r) + J_r \qquad (4)$$

where $J(r)$ is the output, $H(r)$ is a linear transfer function, $X(r)$ is the input, and $J_r$ is a "remnant" term, containing sytem non-linear and random elements. It is incumbent upon the TACS engineer, therefore, to justify assumptions which make $J_r$ negligibly small. These assumptions must deal with environmental factors, human inconsistencies and just simply non-linear subtasks. The first two of these factors can be very subjective, while the last can often be handled by linear approximation techniques.

The easiest way to guarantee linearity is to model the subsystem using only four basic sub-elements: integration, differentiation, algebraic addition and multiplication by a constant [8]. In [6], there are several possible combinations of these building blocks discussed for a variety of human operations in tracking. One of the simplest versions is the classical, first-order, low-pass filter, as shown in Figure 4. Note that the forward gain parameter, $\lambda$, is actually composed of two cascaded gains, one which can be controlled externally and one which cannot. Obviously, an increase in either $n$ or $\gamma$ will result in a decrease in forward gain of the system. For simplicity, the feedback gain has been made unity, although this restriction is removed in the two-parameter discussion in Appendix 2.
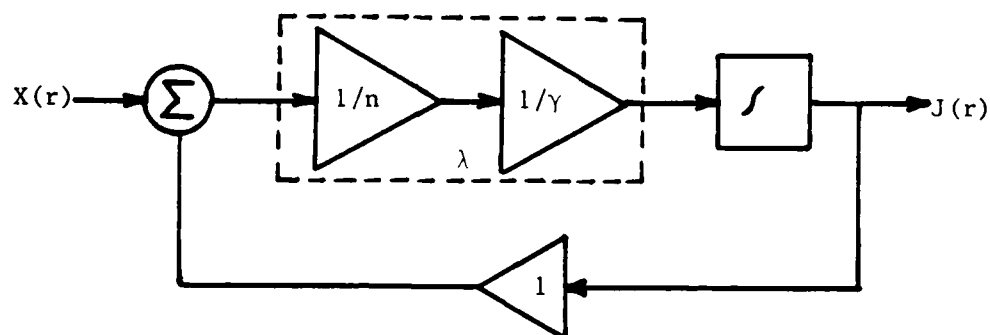
Figure 4.  Simple model of the MCE execution subsystem

We now go to the frequency domain, where the model is easier to use and understand.  Using the variables as indicated in Equation (4) and Figure 4, we have:
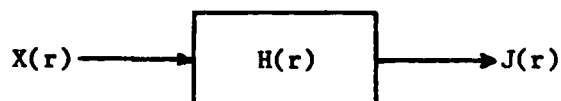


$$H(r) = 1/(j\gamma nr + 1) \tag{5}$$

Disregarding the phase information, we keep only $|H(r)|$.  The sketch of Figure 5(a) shows the low-pass characteristics of $|H(r)|$ and yields two important (and logical) insights into this model:

(1)  It recognizes that at higher frequencies, performance, on an operation by operation basis, declines.

(2)  It incorporates both internal and external parameters which can greatly alter the transfer characteristic.

The transfer function of Figure 5(a) is still difficult to work with, however, so it will be approximated by a different function. Note that the



(a) $\quad |H(r)| = 1 \Big/ \sqrt{(\gamma nr)^2 + 1}$
(b) $\quad G(r) = 2 - e^{\beta nr}$
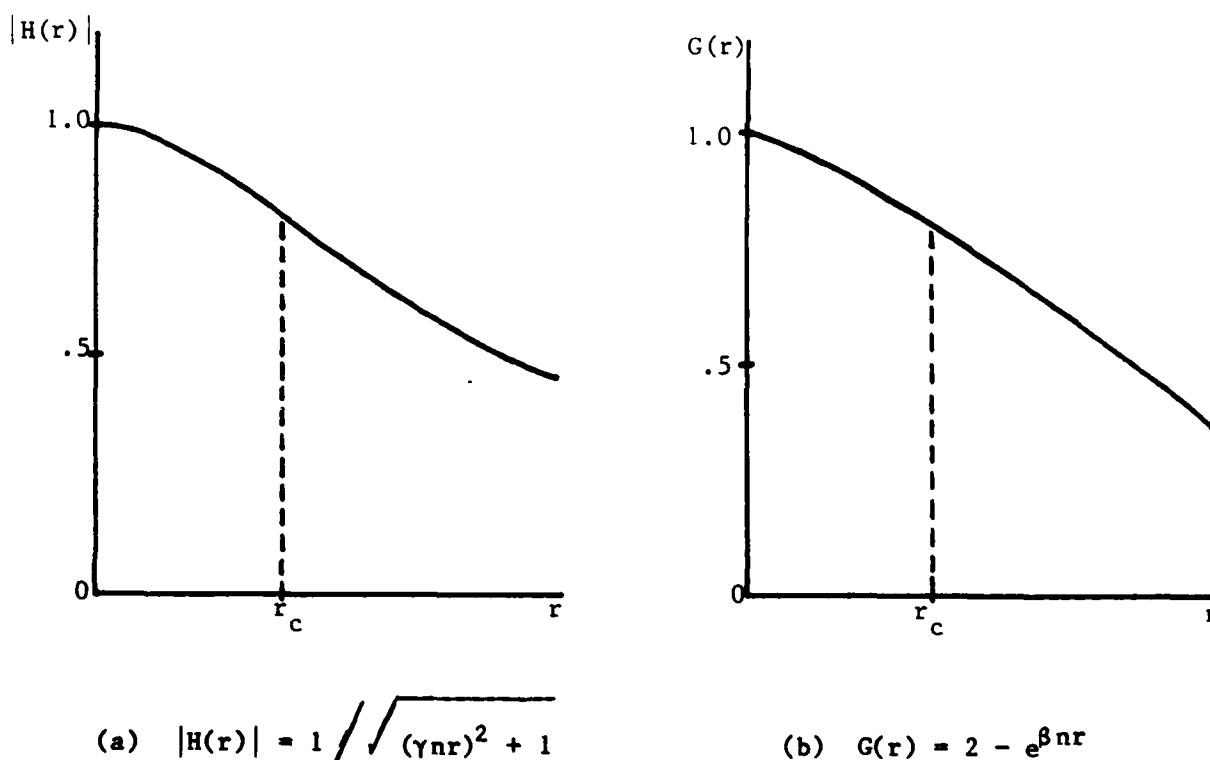
Figure 5. Transfer characteristics of the MCE model

convex portion of $|H(r)|$, in Figure 5(a), is similar to an inverted exponential. In fact, if $|H(r)|$ is truncated at $r_c$, it can be approximated by

$$G(r) = 2 - e^{\beta nr}, \tag{6}$$

where $\beta = k\gamma$, for some constant k. In this form, we have the familiar internal parameter, $\beta$. Figure 5(b) demonstrates how closely the truncated portion of $|H(r)|$ may be approximated by $G(r)$.

The final step in the modeling process is to form $J(r)$, the objective function of MCE and the output of the MCE execution subsystem. Recall from

Section 2.1 that the input to the execution subsystem was simply r, the operating rate. Starting with the general form of J(r), we have:

$$J(r) = |H(r)| \cdot |X(r)|$$

$$J(r) = G(r) \cdot |X(r)|$$

$$= r\,(2 - e^{\beta nr}) \tag{7}$$

Equation (7) is the final form of the model, which will be used for the remainder of the thesis (except Appendix 2). It is convex for all positive values of r, n, and $\beta$, and, if n is treated as a constant (no control), is maximized as follows:

$$J(r)_{max} = J(r*)$$

$$r* = \{r: \ dJ(r)/dr = 0\}$$

$$dJ/dr \bigg|_{r=r*} = 2 - (\beta nr* + 1)\,e^{(\beta nr*)} = 0$$

$$r* = .375/\beta n \tag{8}$$

$$J(r*) = (.375/\beta n)\,(2 - e^{(.375)}) = .204/\beta n \tag{9}$$

Figure 6 compares the objective function, J(r), for different values of n, with $\beta$ fixed, and different values of $\beta$, with n fixed. It also depicts the location of r* for each curve.

(a)  β = 0.15                              (b)  n = 10
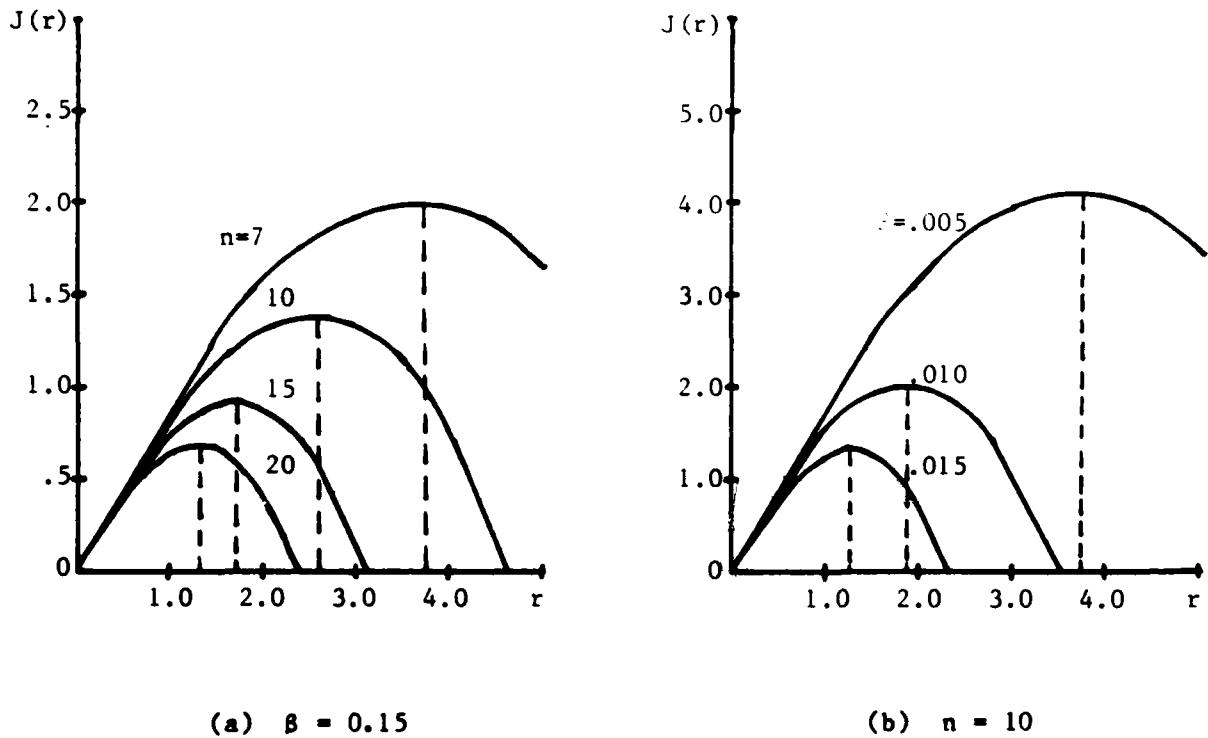
Figure 6.  The ideal MCE objective function

In summary, we have developed a model for the execution subsystem of MCE, which is the frequency domain objective function $J(r)$.  The input, $r$, is a rate value dictated by the command subsystem, which must maximize $J(r)$.  The model contains two parameters:  $\beta$, an internal parameter, which is characteristic of the execution subsystem; and n, the number of customers, which can be affected externally.  We now turn our attention to the manner in which this model may be controlled.

## 4.  THE PIECE-WISE LINEAR APPROACH

A significant problem in the incentive control approach is finding the strategy which will bring about the desired response in the follower.  In [3] and [4] it is seen that if the strategy is made a function of the difference between the actual and desired responses, the follower, when he substitutes the incentive into his own objective function, will be forced to maximize at or near the desired point.

In Section 2.1, the variable n became the leader-follower control variable, as a function $\hat{n}$.  This is the same n as in Equation (7) of the MCE model, and it will be the control for MCE, with TACC, the leader, determining $\hat{n}$.  It is tempting, at first, to use a linear construction of the control:

$$\hat{n} = n_o + k \, (r - r^*), \quad n_o \geq 0, \; k \geq 0 \tag{10}$$

where $n_o$ is a constant term, representing the number of customers MCE will have to handle no matter what (TACC may have no control over $n_o$); r is the independent rate variable; $r^*$ is the solution to Equation (8), given $n_o$ and $\beta'$; and k is a gain parameter which is calculated by TACC.

The problem with Equation (10) is that if $(r - r^*) < 0$, then $\hat{n} < n_o$. It is seen from Figure 6(a) that smaller and smaller values of n lead to larger and larger maximum values of $J(r)$ and allow higher $r^*$.  In fact, if $(r - r^*) = -n_o$, then $\hat{n} = 0$ and MCE has no customers to process and achieves its highest maximum!  It was stipulated, however, that $n_o$ was the minimum number of customers that must be serviced, so we are led to propose a slightly different $\hat{n}$.

### 4.1 The Control Structure

Consider the following control structure:

$$\hat{n} = n_o + k \, |r - r*|. \tag{11}$$

With this function, which shall be called the piece-wise linear incentive strategy, $\hat{n} \geq n_o$ for any r and $k \geq 0$. In view of Figure 6(a), MCE can do no better than processing $n_o$ customers at r*:

$$\hat{n} \Big|_{r=r*} = n_o + k|r* - r*| = n_o \tag{12}$$

Before the effect of this control is investigated, it must be noted that the piece-wise linear approach does not lead to a continuously differentiable objective function in r, but contains a discontinuous slope at $r = r*$ (see Figure 7(c)). Therefore, Equation (3) cannot be used to find r*. Instead, the slope of the objective function must be calculated on each side of the "peak" at r*, to ensure that the largest value of $J(r)$ occurs at that peak.

Suppose TACC announces a strategy of the form in Equation (11). The MCE commander, in order to effectively maximize his objective function, must substitute $\hat{n}$ for n in Equation (7):

$$J(r) = r \, (2 - e^{\beta \hat{n} r})$$
$$= r \, (2 - e^{\beta [n_o + k|r-r*|] \, r}) \tag{13}$$

Now equation (13) must be analyzed for two intervals of the r-axis: $r \geq r*$ and $0 < r \leq r*$.

### 4.1.1 Case I: $r \geq r^*$

Statement: Suppose $n_o$ is known and $\beta'$ is estimated by TACC, so that $r^*$ can be calculated by Equation (8). Suppose also that the minimum expected value of $\beta$, denoted $\beta_1$, is known. Then a sufficient condition for $J(r)$ to have a negative slope on the interval $[r^*, \infty]$ is to choose a $k$ satisfying:

$$k > \frac{2 - (\beta_1 n_o r^* + 1)\, e^{(\beta_1 n_o r^*)}}{\beta_1 r^{*2}\, e^{(\beta_1 n_o r^*)}} \tag{14}$$

Proof: The objective function, for this interval, can be written:

$$J(r) = r\,(2 - e^{\beta[n_o + k(r - r^*)]r}) \tag{15}$$

If $J(r)$ has a negative slope for all $r$ in $[r^*, \infty]$ and $\beta$ in a given range, $\beta > \beta_1$, then its maximum value will occur at $r^*$. This can be stated mathematically:

$$dJ/dr < 0$$

$$2 - [\beta\,(\hat{n} + d\hat{n}/dr)\,r + 1]e^{(\beta \hat{n} r)} < 0$$

$$d\hat{n}/dr = k, \text{ based on Equation (15),}$$

$$\text{so } 2 - (\beta \hat{n} r + 1)\, e^{(\beta \hat{n} r)} - k\beta r^2\, e^{(\beta \hat{n} r)} < 0 \tag{16}$$

The worst case for ensuring Inequality (16) is if $\beta$ and $r$ are at their minimum values. Therefore, let $\beta = \beta_1$, $r = r^*$, and thus $\hat{n} = n_o$. Inequality (16) becomes:

$$2 - (\beta_1 n_o r^* + 1)\, e^{(\beta_1 n_o r^*)} - k\beta_1 r^{*2}\, e^{(\beta_1 n_o r^*)} < 0$$

Therefore, $\qquad k > \dfrac{2 - (\beta_1 n_o r^* + 1)\ e^{(\beta_1 n_o r^*)}}{\beta_1 r^{*2}\ e^{(\beta_1 n_o r^*)}}$

Therefore, when TACC announces this strategy, he will have to select k based on Inequality (14). However, there is another side of the function which must be considered.

## 4.1.2  Case II:  $0 < r \le r^*$

This case is a little more difficult to analyze. Normally, it would be sufficient to guarantee a positive slope at $r^*$, but if k is made large enough, a cubic-like oscillation (maximum followed by a minimum) is induced on $[0, r^*]$. Therefore, k must be large enough to provide a positive slope at $r^*$, but not so large as to induce an oscillation between 0 and $r^*$.

> Statement: Suppose $n_o$ is known and $\beta'$ estimated by TACC, so $r^*$ can be calculated. Suppose that the maximum expected value of $\beta$, denoted $\beta_2$, is also known. Then a sufficient condition for $J(r)$ to have a positive slope on the interval $[0, r^*]$ is to choose a k satisfying:

$$k > - \frac{2 - (\beta_2 n_o r^* + 1)\ e^{(\beta_2 n_o r^*)}}{\beta_2 r^{*2}\ e^{(\beta_2 n_o r^*)}} \qquad (17)$$

> provided there is no real solution, in the interval $[0, r^*]$, to the equation $dJ/dr = 0$.

Proof:  For this interval, the objective function becomes:

$$J(r) = r(2 - e^{\beta[n_o + k(r^*-r)]r}) \qquad (18)$$

As mentioned earlier, the first task is to guarantee a positive slope at $r = r^*$, with $\beta$ in a given range $(\beta < \beta_2)$:

$$dJ/r > 0 \text{ and } d\hat{n}/dr = -k$$

$$2 - (\beta \hat{n} r + 1) \, e^{(\beta \hat{n} r)} + k\beta r^2 \, e^{(\beta \hat{n} r)} > 0. \tag{19}$$

Finally, making the substitutions $r = r*$, $\beta = \beta_2$ and $\hat{n} = n_o$:

$$2 - (\beta_2 n_o r* + 1) \, e^{(\beta_2 n_o r*)} + k\beta_2 r*^2 \, e^{(\beta_2 n_o r*)} > 0 \ .$$

Therefore, $\qquad k > - \dfrac{2 - (\beta_2 n_o r* + 1) \, e^{(\beta_2 n_o r*)}}{\beta_2 r*^2 \, e^{(\beta_2 n_o r*)}}$

In summary, if TACC decides to use the piece-wise linear approach, he must calculate a minimum value of k for both sides of the slope discontinuity at r*. He then must select a k which satisfies both cases but is not so large that a maximum-minimum pair forms on [0,r*]. The following example will help greatly in understanding the preceding discussion.

## 4.2  An Example

Suppose that TACC estimates that MCE should have an internal parameter $\beta' = .01$, and it is given that $n_o = 10$. Figure 7(a) depicts the ideal MCE objective function (without control). The r* corresponding to the maximum is found by Equation (8):

$$r* = .375/\beta' n_o = .375/(.01)(10) = 3.75$$

It is assumed that TACC's estimate of $\beta'$ is correct, but that MCE commander perceives a different internal parameter, $\beta = .008$. Therefore, MCE will use a different objective function to maximize, which is depicted in Figure 7(b). Because of this, he will actually drive the MCE at some $\tilde{r} \neq r*$,

where

$$\tilde{r} = .375/(.008)(10) = 4.69$$

When TACC observes MCE operating at this unexpected rate, he may wish to intervene, using the piece-wise linear approach.

First, TACC must estimate the range in which $\beta$ falls. Suppose the estimated range is $.006 \leq \beta \leq .012$.

Case I: $r \geq 3.75$

$$k > \frac{2 - (\beta_1 n_o r^* + 1)\, e^{(\beta_1 n_o r^*)}}{\beta_1 r^{*2}\, e^{(\beta_1 n_o r^*)}}$$

and if $\beta_1 = .006$, $n_o = 10$, and $r^* = 3.75$, then $k > 4.41$    (20)

Case II: $0 < r \leq 3.75$

$$k > -\frac{2 - (\beta_2 n_o r^* + 1)\, e^{(\beta_2 n_o r^*)}}{\beta_2 r^{*2}\, e^{(\beta_2 n_o r^*)}}$$

and $\beta_2 = .012$, so $k > 1.04$.    (21)

Therefore, in order to satisfy both Inequalities (20) and (21), TACC decides to pick $k = 5$. The incentive strategy that he announces is:

$$\hat{n} = 10 + 5|r - 3.75|.$$    (22)

Once MCE substitutes Equation (22) into his objective function, his new curve will appear as in Figure 7(c). Note that the function must be maximized at $r = r^* = 3.75$ and that no cubic oscillations occur on $[0, r^*]$. Figure 7(d), however, demonstrates how a large enough gain ($k = 17$) can induce the maximum-minimum pair.
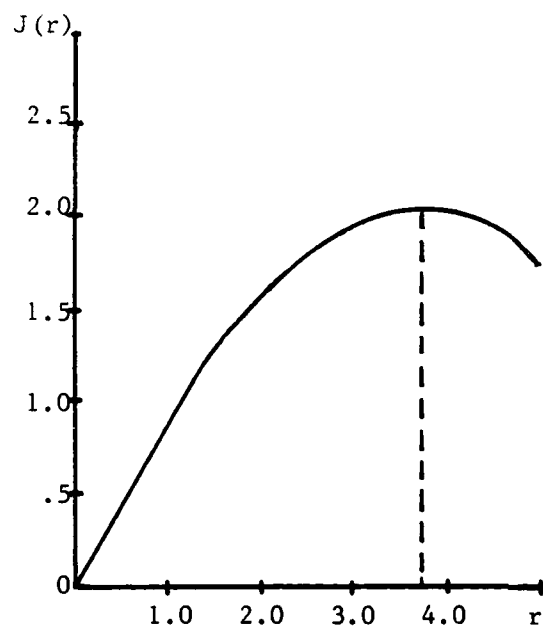
(a)  $J(r) = r (2 - e^{.1r})$
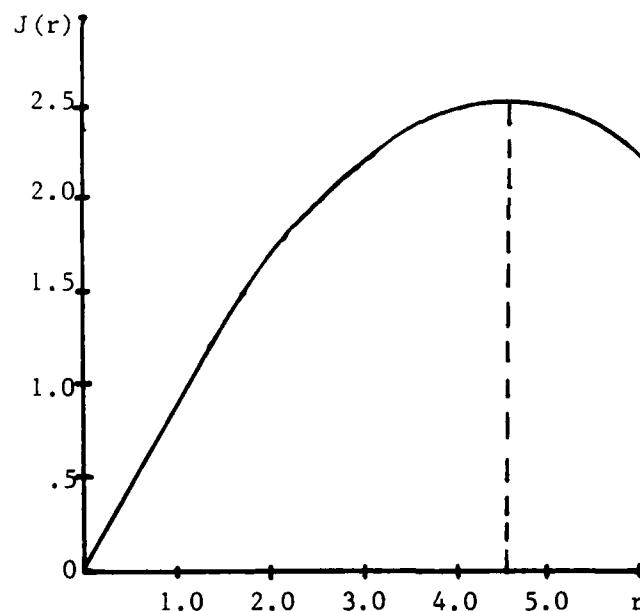
(b)  $J(r) = r (2 - e^{.08r})$
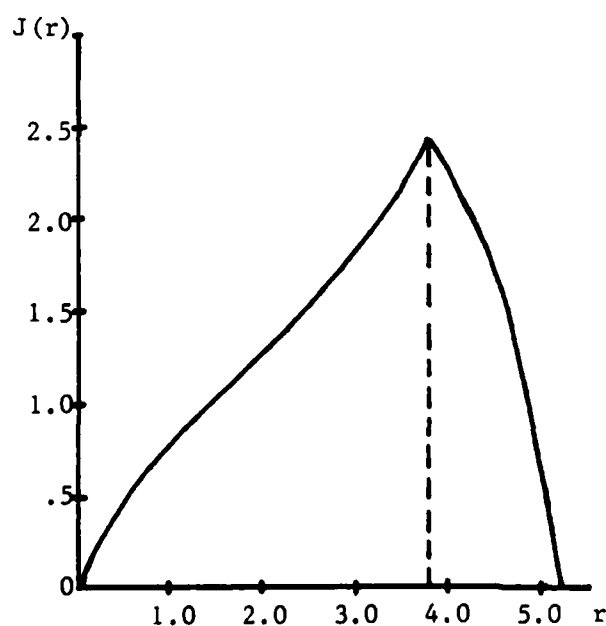
(c)  With control:  $k = 5$
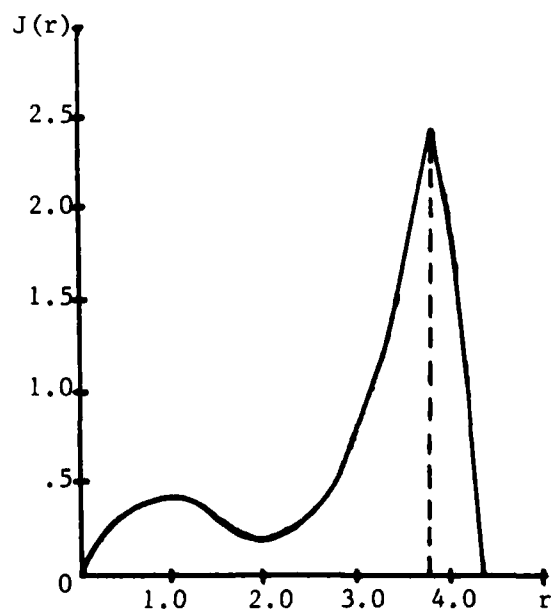
(d) With control:  $k = 17$

Figure 7.  Objective function with piece-wise linear control

It is important to remember that although $\beta = .008$ is used to generate curves (b), (c), and (d) in Figure 7, any $\beta$ in the interval [.006,.012] would have yielded the same r*, although the shapes of the curves and the resulting $J_{max}$ values would vary. Because of this, r* is considered insensitive to changes in $\beta$, for the range $.006 \leq \beta \leq .012$.

## 5. THE QUADRATIC APPROACH

### 5.1 The Control Structure

There is an alternative approach to controlling the MCE, and that is to use a control variable of the form:

$$\hat{n} = n_o + k_1 (r - r*)^2 + k_2 (r - r*) \tag{23}$$

In this case, $n_o$ and $r*$ are the same as in the piece-wise linear approach, but now two gain constants, $k_1$ and $k_2$, have been introduced. One of the advantages of this structure is that $\hat{n}$ is a continuous function, so MCE's objective function will also be continuous when this control is substituted for n. Therefore, $J(r)$ does not have to be evaluated for two different cases, as in the former approach, and $r*$ can be found using Equation (3):

$$r* = \{r: \quad dJ/dr = 0\} \tag{24}$$

$$dJ/dr = 2 - \{\beta r[(d\hat{n}/dr)r + \hat{n}] + 1\} \, e^{(\beta \hat{n} r)} = 0 \tag{25}$$

where $d\hat{n}/dr = 2k_1(r - r*) + k_2$. Therefore, by substitution:

$$dJ/dr = 2 - \{\beta r[(2k_1(r-r*) + k_2)r + (n_o +$$

$$k_1(r-r*)^2 + k_2(r-r*))] + 1\} \, e^{(\beta \hat{n} r)} = 0 \tag{26}$$

The solution of Equation (26) is not a trivial matter, and we found it convenient (and very accurate) to use the following procedure:

$$dJ/dr = 2 - [f_2(r) + f_1(r) + 1] \, e^{(\beta \hat{n} r)} = 0$$

where $f_1(r)$ and $f_2(r)$ are determined from Equation (26). Thus

$$r = \{\ln(2) - \ln[f_2(r) + f_1(r) + 1]\}/\beta n$$

Using a first order Taylor series approximation of the second logarithmic term about r*, the problem eventually reduces to a cubic equation, which is solved by computer using the cubic formula.

Calculation of the constants $k_1$ and $k_2$ by TACC is relatively easy, however, if we remember that TACC's goal is to have Equation (26) satisfied at r=r*. By substituting r* into Equation (26), it is seen that most terms drop out, and the remaining equation is:

$$2 - (\beta r*[k_2 r* + n_o] + 1) \, e^{(\beta n_o r*)} = 0$$

$$k_2 = \frac{2 - (\beta n_o r* + 1) \, e^{(\beta n_o r*)}}{\beta r*^2 \, e^{(\beta n_o r*)}} \tag{27}$$

It is interesting to note that Equation (27) depends only on $k_2$. This is significant in that if TACC knew exactly which $\beta$ MCE was going to assume in its maximization, he could simply calculate $k_2$ by Equation (27) and use any $k_1$, with the resulting objective function maximized at r*.

In reality, however, $\beta$ is not known by TACC, and it is desired to minimize the sensitivity of r* to fluctuations in $\beta$. Again, it is assumed that $\beta$ can be confined to a range, such as $\beta_1 \leq \beta \leq \beta_2$. Additionally, an average $\beta$ for the range, $\beta_c$, must be selected by TACC. Equipped with this data, TACC can methodically select $k_1$ and $k_2$ so that if MCE uses $\beta_c$, he will maximize exactly at r*, and if he uses some other $\beta$ in $[\beta_1,\beta_2]$, his maximization will remain within a given tolerance of r*. One additional advantage of this policy is that $\beta'$, the value which TACC thinks actually characterizes MCE and upon which r* is based, does not have to be in the interval $[\beta_1, \beta_2]$. If $\beta'$ does fall in $[\beta_1,\beta_2]$, however, it is convenient to let $\beta_c = \beta'$ for analysis and calculation purposes.

The procedure for finding $k_1$ and $k_2$ is as follows:

(1) Define the anticipated range of $\beta$, by selecting $\beta_1$, $\beta_2$ and $\beta_c$.

(2) Compute $r^*$ for the ideal objective function and specify the allowable tolerance in r, the operating rate of MCE, about $r^*$. This is also expressed as an interval, $[r_1, r_2]$, containing $r^*$.

(3) Using $n_o$, $r^*$ and $\beta_c$, calculate $k_2$ using Equation (27).

(4) Using equation (26) and the above computed value of $k_2$, find $k_1$ at each boundary of $[r_1, r_2]$:

    a. At $r_1$, use $\beta_2$

    b. At $r_2$, use $\beta_1$

    The final $k_1$ that TACC selects must be larger than the largest $k_1$ computed for both of the above cases.

There is one more consideration in selecting $k_1$: if $k_1$ is too large, the induced MCE objective function will have a very sharp "peak" at its maximum. This could mean that even if MCE is maximizing within $[r_1, r_2]$ tolerance, the maximization point, $\tilde{r}$, may still be far enough away from $r^*$ to cause a substantial increase in $\hat{n}$. Recall that $\hat{n}$ is based on deviations in r from the ideal $r^*$. If $k_1$ is large enough, even a small deviation in r will cause $\hat{n}$ to be enforced (i.e., extra customers are given to MCE for not "cooperating"). This is inconsistent, however, with the fact that MCE is within tolerance.

The solution to this dilemma lies in analyzing the "discrete" nature of $\hat{n}$. Since TACC cannot give fractions of customers to MCE, only integral values of $\hat{n}$ are permitted. Therefore, TACC must wait until $\hat{n}$ increases sufficiently over $n_o$ to actually increment with an additional customer

(enforce).  For this thesis, we assume that if $\hat{n} = n_o + .5$, the control will be enforced.  This discussion leads us to a fifth step:

(5) Compute $k_{1_{max}}$, such that for r in $[r_1, r_2]$, $\hat{n} < n_o + .5$.  This last step may be accomplished precisely by evaluating Equation (23) at $r_1$ and $r_2$ or by using a graph of $\hat{n}$ vs. r, as in Figure 8(c).

## 5.2 An Example

Again, suppose $n_o = 10$ customers and TACC uses $\beta' = .01$ as the actual parameter of MCE.  It is anticipated, however, that MCE will choose a $\beta$ in the interval $.007 \leq \beta \leq .015$.  For simplicity, TACC uses $\beta_c = .01$.  Suppose further that the maximum allowable tolerance in $\tilde{r}$ is $3.6 \leq r \leq 3.9$ (recall that r* for these data is 3.75).  Proceeding from the third step:

$$(3) \qquad k_2 = \frac{2 - (\beta_c n_o r^* + 1)\, e^{(\beta_c n_o r^*)}}{\beta_c r^{*2}\, e^{(\beta_c n_o r^*)}}$$

$$= \frac{2 - [(.01)(10)(3.75) + 1]\,(1.455)}{(.01)(3.75)^2(1.455)}$$

$$= 0$$

(4) At $r_1 = 3.6$, $\beta_2 = .015$.  Since $k_2 = 0$, using Equation (26):

$$k_1 = 7.1$$

At $r_2 = 3.9$, $\beta_2 = .007$.  With $k_2 = 0$ and Equation (26):

$$k_1 = 7.5$$

Therefore, $k_1$ must be greater than 7.5.

(5)   Since $k_2 = 0$, Equation (25) becomes:

$$\hat{n} = 10 + k_1(r - r*)^2$$

Evaluating at $r_1 = 3.6$:

$$\hat{n} = 10 + k_1(3.6 = 3.75)^2 < 10.5$$

$$k_1 < 22.2$$

Because $k_2 = 0$, Equation (23) is symmetric about r* and the same result is obtained for $r_2 = 3.9$. Therefore $k_{1_{max}} = 22.2$.

Thus, TACC must select $k_2 = 0$ and $k_1$ in the range [7.5, 22.2]. Suppose he chooses $k_1 = 10$. The control that is announced by TACC to MCE is:

$$\hat{n} = 10 + 10(r - 3.75)^2 \tag{28}$$

Finally, for this example, assume that MCE is actually using $\beta = .008$ in its operation. After substitution of Equation (28) in the MCE objective function, the MCE must maximize:

$$J' = r(2 - e^{.008[10 + 10(r - 3.75)^2]r}) \tag{29}$$

Figure 8 is an illustration of this example. In Figure 8(a), the MCE objective is depicted for $\beta = .008$ without control. Note that the maximum occurs at $\tilde{r} = 4.69$. In Figure 8(b), the effect of the control is seen as Equation (29) is displayed. MCE maximizes at $r = 3.82$, which is well within tolerance. Figure 8(c) is a plot of $\hat{n}$ vs. r, about r* = 3.75, for different values of $k_1$ ($k_2 = 0$). The superimposed rectangle is the region in which $\hat{n}$ is essentially $n_o$, because of the discrete nature of the control. Figure 8(c) may serve as a "look-up table" for determining the

maximum value of $k_1$ (step 5 in the procedure). The purpose of Figure 8(d) is to demonstrate the "centering" properties of $k_2$. The data are the same as for Figure 8(b), except the objective function is sketched for $k_2 = 5$ and $k_2 = -5$. If TACC knew that $\beta = .008$ in advance, the actual centering value of $k_2$ could, of course, be calculated from Equation (27):

$$k_2 = \frac{2 - [(.008)(10)(3.75) + 1] (1.35)}{(.008)(3.75)^2(1.35)} = 1.613$$
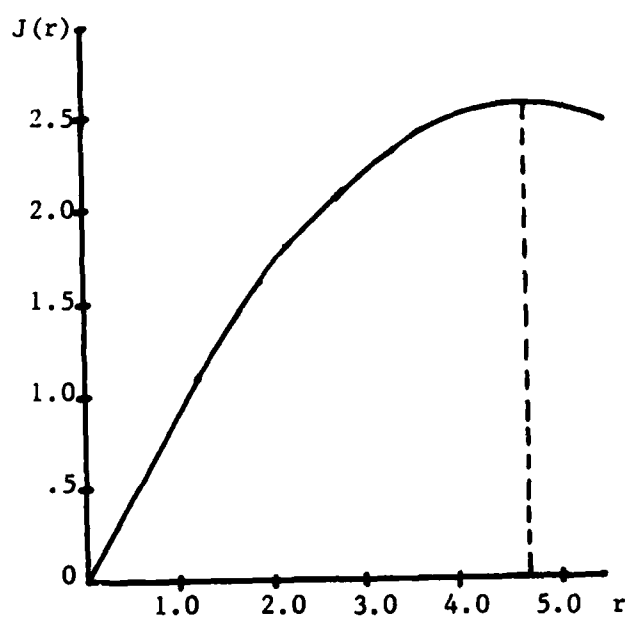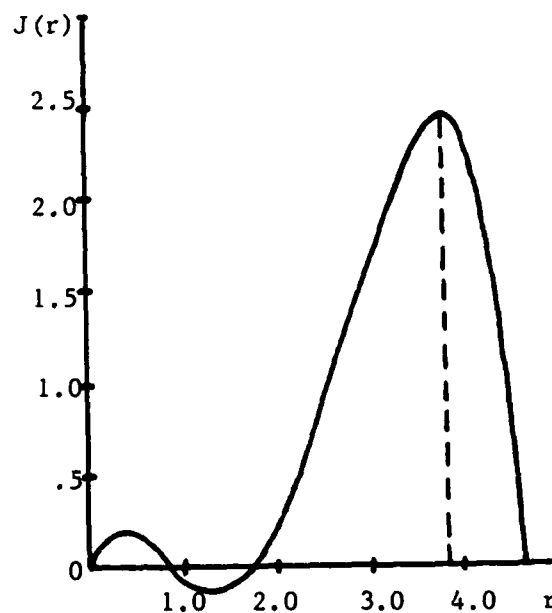


(a) No control: $\beta = .008$      (b) With control: $k_1 = 10$, $k_2 = 0$

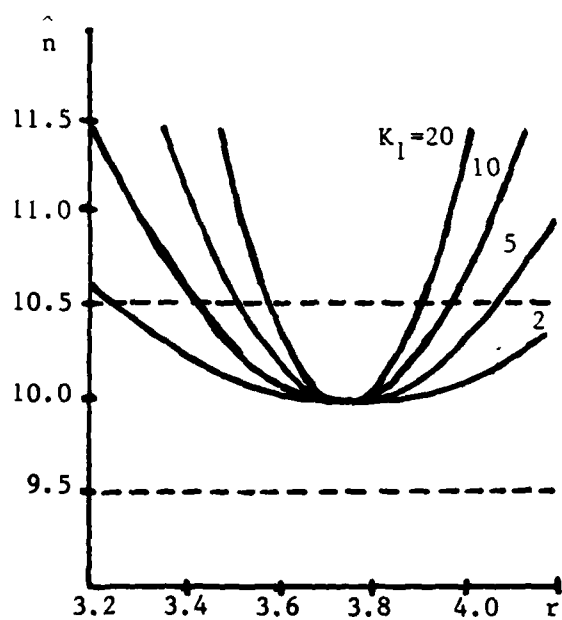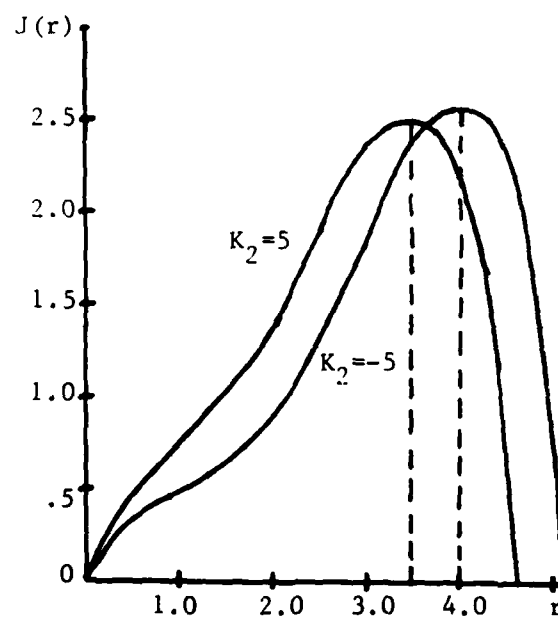Figure 8. Objective function with quadratic control

(c) $\hat{n}$ vs. r

(d) "Centering" property of $k_2$

Figure 8. Continued.

# 6. CONCLUSION

In this thesis, we have developed the mathematical framework for employing incentive control in the Tactical Air Control System. We modeled MCE as being composed of a command subsystem, whose function is to maximize its objective function, and an execution subsystem, which behaves like a transfer function, whose characteristics are governed by an internal and an external parameter.

The goal of the incentive control strategies, introduced in Chapters 4 and 5, is to ensure that MCE maximizes its objective function at or near a desired operating point, r*, regardless of the value of the internal parameter MCE uses in its objective function. The piece-wise linear control strategy makes r* _insensitive_ to changes in $\beta$ over a predetermined range of $\beta$, but is computationally more difficult to analyze. The quadratic approach is somewhat easier to analyze and has the advantage that $\beta'$ does not have to be in $[\beta_1, \beta_2]$. It is not, however, totally insensitive to changes in $\beta$ but can be made _minimally_ sensitive for given tolerances in MCE response. Both controls have the distinct advantage that for any model selected, if the objective function is a monotonically decreasing function of the control variable n (reference Figure 6), the induced objective function will be maximized only at the bias value, $n_o$ (which corresponds to r*).

Another important application of incentive mechanisms in the $C^3$ setting is in Command, Control, and Communications Countermeasures ($C^3CM$). In this context, the participants are viewed as non-cooperative. The use of incentive strategies by one superpower to induce "cooperation" by another superpower in strategic arms control is discussed in [4]. These

same ideas can be extended to the tactical theater to force an enemy $C^3$ system to behave in a certain manner. The steps taken are exactly the same as outlined in this thesis:

(1) Model the adversary's operation and determine his objective function.

(2) Determine all unknown internal parameters of the objective function.

(3) Determine what parameter(s) in the objective function can be affected by an incentive strategy.

(4) Design an appropriate strategy.

(5) Announce the strategy to the adversary.

APPENDIX 1

SENSITIVITY FUNCTION ANALYSIS

Throughout this thesis, our discussion has been based on the fact that MCE might choose some value of $\beta$ other than $\beta'$ in its objective function. At the same time, our assumption has been that $\beta'$ (the value that TACC thinks MCE should have) is actually the more appropriate value. We desire, then, to have some measure of how close $J_{max}$, the _actual_ maximum value of the MCE output, is to $J'_{max}$, the ideal calculated by TACC.

In [4], sensitivity functions are introduced to provide such a measure. Using the derivative terms of the Taylor series expansion of J about $\beta'$, when $r = r*$, we denote the first order sensitivity function:

$$
I_1 = dJ(n,r,\beta)/d\beta \Big|_{\substack{\beta=\beta' \\ r=r* \\ n=n_o}}
$$

$$
= [(dJ/dn)(dn/dr) + dJ/dr] \, (dr/d\beta) \Big|_{\substack{\beta=\beta' \\ r=r* \\ n=n_o}}
$$

(30)

We first analyze each term of Equation (30) in view of the piece-wise linear approach. $dJ/dn$ will always be negative, due to the monotonic relationship between J and n in the original model. $dn/dr$ is discontinuous at $r = r*$, since it is equal to k, for $r > r*$, and $-k$, for $r < r*$. $dr/d\beta$ is zero, however, because in the piece-wise approach, we have total insensitivity of r to changes in $\beta$ in $[\beta_1,\beta_2]$, which contains $\beta'$. Due to this fact, the entire function is zero.

For the quadratic approch, we can never make $dr/d\beta$ equal to zero but can only minimize it. $dJ/dr$ is still zero, of course, and $dJ/dn$ is still

always negative. The easiest way to make $I_1 = 0$, then, is to ensure $dn/dr = 0$. Recalling the discrete nature of $\hat{n}$, we can say that for a small range of r about r*, the change in $\hat{n}$ with respect to r is zero. Furthermore, since our selection of $k_1$ was based on a range of $\beta$, if $\beta'$ is included in $[\beta_1, \beta_2]$, then $dn/dr = 0$ over that range (in a "discrete" sense). If $\beta'$ is not included in $[\beta_1, \beta_2]$, we cannot use this function as a measure of sensitivity about $\beta'$.

If $I_1 = 0$, the dominant term of the expansion becomes the second order sensitivity function, which we define as:

$$I_2 = d^2 J(n,r,\beta)/d\beta^2 \bigg|_{\substack{\beta=\beta' \\ r=r* \\ n=n_o}}$$

$$= r_\beta{}^2 (n_r{}^2 J_{nn} + n_r J_{nr} + n_r J_{rn} + J_{rr})$$

$$+ (J_n n_r + J_r) r_{\beta\beta} \bigg|_{\substack{\beta=\beta' \\ r=r* \\ n=n_o}} \tag{31}$$

Again analyzing the piece-wise linear approach first, we find that since $dr/d\beta = 0$ at $\beta = \beta'$, the entire first term is zero. Total insensitivity of r to $\beta$ in $[\beta_1, \beta_2]$ also guarantees that $r_{\beta\beta} = 0$, thus $I_2 = 0$ for the piecewise approach. In fact, all higher order derivatives of J will also vanish in a neighborhood of $\beta'$, so for this approach the maximum value of the objective function is totally <u>insensitive</u> to changes in $\beta$ in $[\beta_1, \beta_2]$.

For the quadratic approach, since $dJ/dr = 0$ and $dn/dr = 0$, Equation (31) reduces to the following expression:

$$I_2 = (dr/d\beta)^2 (d^2 J/dr^2) \tag{32}$$

We know that $dr/d\beta$ cannot be zero but is minimized for $k_1$ very large.
Therefore, the first product term can only be a small positive number.
Also, the objective function is convex in r for all $\beta$ and n, so the second
product term is always a negative valued function. Therefore, to minimize
$I_2$, we should make $k_1$ as large as possible, but not larger than $k_{1_{max}}$
calculated in Chapter 5 (Equation 27).

## APPENDIX 2

## A TWO-PARAMETER MODEL

In Section 3.2, a model was developed for MCE, which included one internal parameter, $\beta$. Now we investigate the effect of incorporating a second parameter, $\alpha$, into the objective function. Consider a transfer function of the form:

$$G(r) = \alpha - e^{(\beta n r)} \qquad (33)$$

The difference between Equation (33) and the one-parameter case corresponds roughly to changing the unity feedback gain of Figure 4 to a variable feedback. The effect of introducing $\alpha$, for $\alpha \leq 2$, is to vary the overall amplitude of the transfer function. The resulting objective function for MCE is:

$$J(r) = r(\alpha - e^{(\beta n r)}) \qquad (34)$$

Examining Equation (34) for the piece-wise linear approach poses no special problems. We again divide the r-axis into two regions and compute dJ/dr for two cases.

## Case I: $r \geq r^*$

Rewriting Equation (18) for the more general case involving $\alpha$:

$$\alpha - (\beta n r + 1)e^{(\beta n r)} - k\beta r^2 e^{(\beta n r)} < 0 \qquad (35)$$

Now, in addition to the worst case values of $\beta$, r, and n, the largest value of $\alpha$ also must be selected, to ensure Inequality (35). Thus, we use $\alpha = 2$. The result for this case, then, is the same as for the one-parameter case:

$$k > \frac{2 - (\beta_1 n_o r^* + 1)e^{(\beta_1 n_o r^*)}}{\beta_1 r^{*2} e^{(\beta_1 n_o r^*)}} \tag{36}$$

## Case II: $0 \leq r \leq r^*$

The same procedure is followed here as in Section 4.1.2, except that the __minimum__ anticipated value of $\alpha$ is used in calculation of k. Denoting this value of $\alpha$ as $\alpha_m$:

$$k > \frac{\alpha_m - (\beta_2 n_o r^* + 1)e^{(\beta_2 n_o r^*)}}{\beta_2 r^{*2} e^{(\beta_2 n_o r^*)}} \tag{37}$$

The problem of inducing the extra oscillation, for large k on $[0, r^*]$, is even more acute for smaller values of $\alpha$. In fact, for $\alpha < 1.5$, there may not exist a k which satisfies Inequality (37) but which avoids the maximum-minimum phenomenom. Therefore, for this model, $\alpha$ should be restricted to the interval $[1.5, 2.0]$.

Analysis of the two-parameter case is likewise straightforward for the quadratic approach. The steps taken are identical to the procedure outlined in Chapter 5, except that an anticipated $\alpha$ range, $\alpha_1 \leq \alpha_c \leq \alpha_2$, is identified as well as the $\beta$ range. Then, the "centering" property of $k_2$ at $\alpha_c$, $\beta_c$ is retained and $k_1$ is selected for a predetermined r tolerance, evaluated at the boundaries $r_1$, $\alpha_1$, $\beta_2$ and $r_2$, $\alpha_2$, $\beta_1$.

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